CASCADE for eA

Building uPDF's using the GBW dipole and the CCFM evolution

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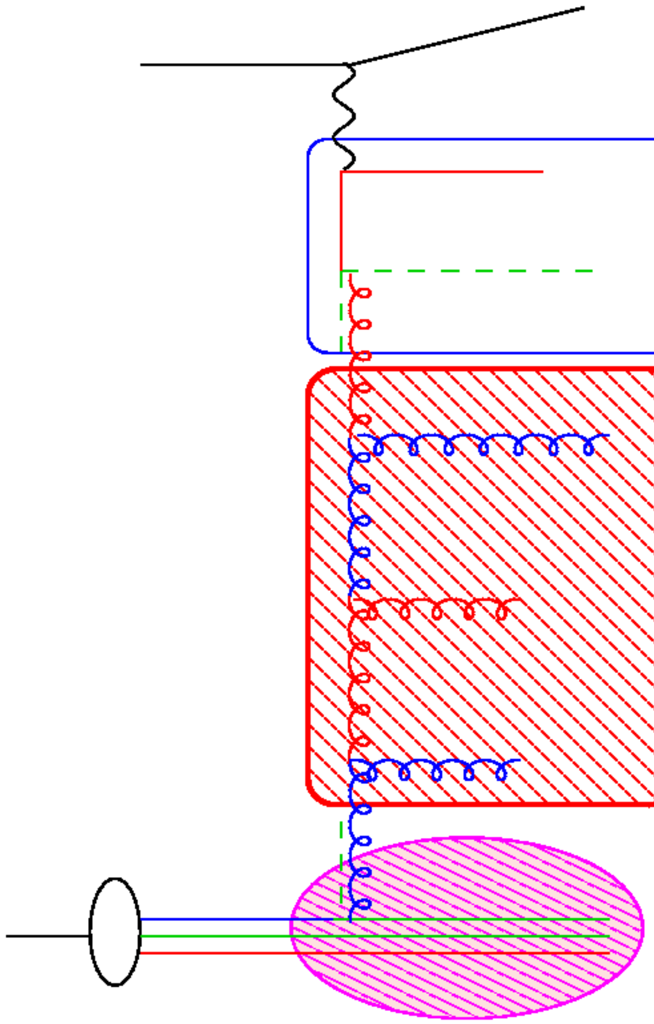
EIC TF 24/5 12

Key eA measurements at an EIC

Deliverables	Observables	What we learn	Stage-I	Stage-II
Integrated gluon momentum distributions	$F_{2,L}$	Nuclear wave function; saturation, Q_s	Gluons at $10^{-3} \lesssim x \lesssim 1$	Exploration of saturation regime
k_T-dependent gluons; gluon correlations	Di-hadron correlations	Non-linear QCD evolution/universality	Onset of saturation; Q_s	Renormalization group evolution
Spatial gluon distributions; gluon correlations	Diffractive vector mesons and DVCS	Small-x non-linear evolution; saturation dynamics	Moderate x with nuclei	Smaller x, saturation

CASCADE - C$_{\text{atani}}$C$_{\text{iafaloni}}$F$_{\text{iorani}}$M$_{\text{archesini}}$ evolution

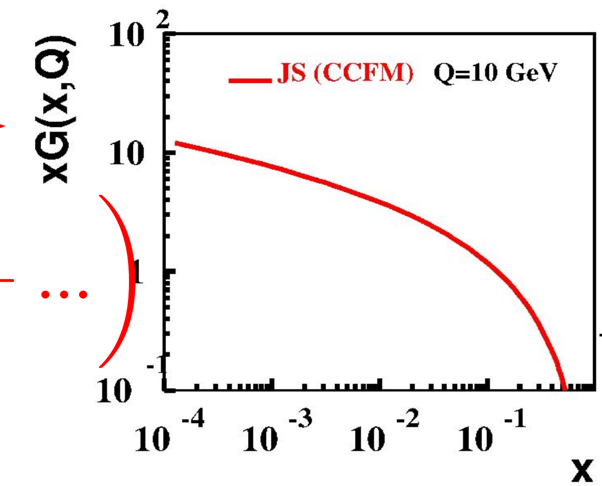


BGF matrix element
off shell

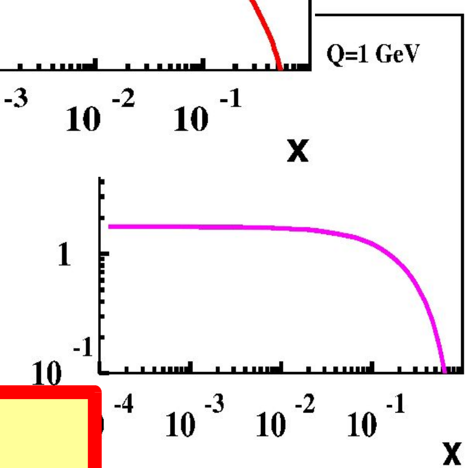
CCFM (all loops)
- angular ordering
- non - Sudakov Δ_{ns}

evolution of parton cascade:

$$\tilde{P} = \bar{\alpha}_s \left(\frac{1}{1-z} + \frac{1}{z} \Delta_{ns} + \ldots \right)$$



initial distribution
~ flat

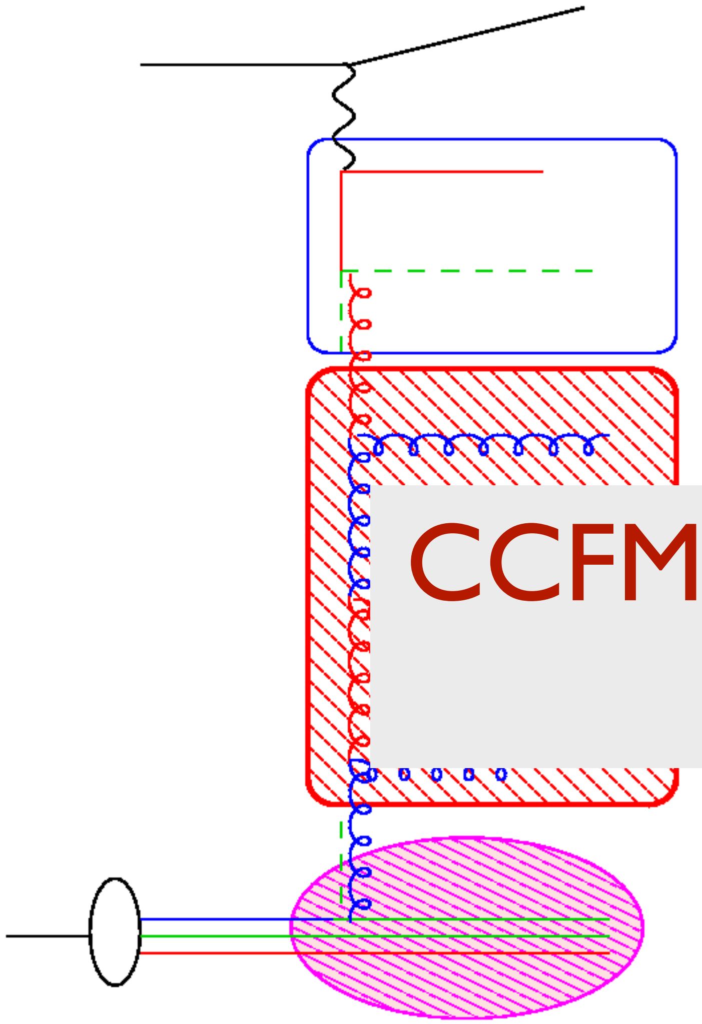


$$\sigma(ep \to e'q\bar{q}) = \int \frac{dy}{y} d^2Q \frac{dx_g}{x_g} \int d^2k_t \hat{\sigma}(\hat{s}, k_t, Q) x_g \mathcal{A}(x_g, k_t, \bar{q})$$

$$\int d^2k_t x_g \mathcal{A}(x_g, k_t, \bar{q}) \simeq x_g G(x_g, Q^2)$$

Slide from H. Jung

CASCADE - C$_{atani}$ C$_{iafaloni}$ F$_{iorani}$ M$_{archesini}$ evolution



BGF matrix element
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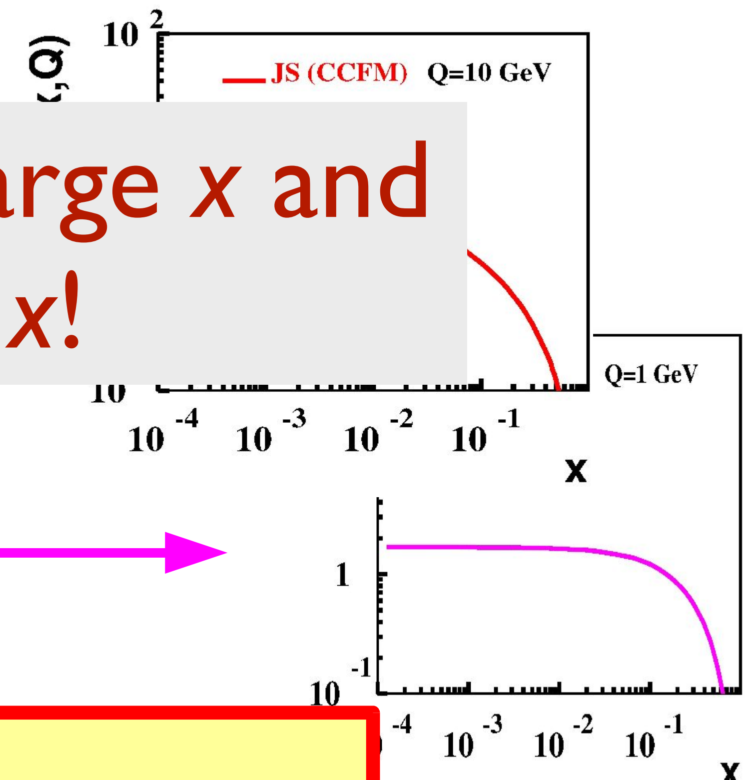


CCFM is DGLAP-like for large *x* and BFKL-like for small *x*!

$$\sigma(ep \to e'q\bar{q}) = \int \frac{dy}{y} d^2Q \frac{dx_g}{x_g} \int d^2k_t \hat{\sigma}(\hat{s}, k_t, Q) x_g \mathcal{A}(x_g, k_t, \bar{q})$$

$$\int d^2k_t x_g \mathcal{A}(x_g, k_t, \bar{q}) \simeq x_g G(x_g, Q^2)$$

Status of CASCADE

Only now that HERA F_2 data is precise enough to answer detailed questions about the CCFM evolution important for the uPDF parametrisation fit!

CCFM evolution for gluon

Slide from H. Jung
DIS2012

- Gluon evolution:

$$x\mathcal{A}(x,k_t,p) = x\mathcal{A}_0(x,k_t,p) + \int \frac{dz}{z}\int \frac{dq^2}{q^2}\,\Theta(p-zq) \times \Delta_s(p,zq)P(z,k_t)\; x\mathcal{A}\left(\frac{x}{z}, k_t+(1-z)q, q\right)$$

- Splitting function: only singular terms

$$P_{gg}(z,k_\perp) = \bar{\alpha}_s\left[\frac{1}{z}\Delta_{ns} + \frac{1}{1-z}\right]$$

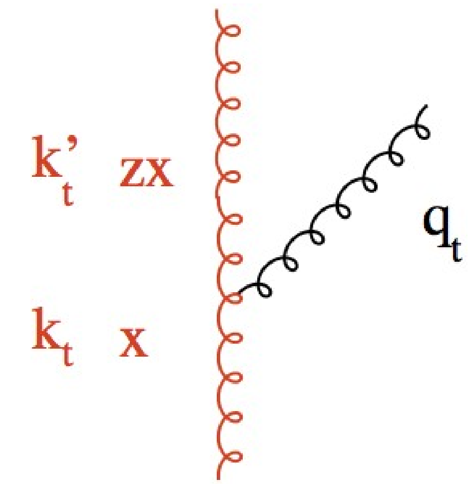
or including non-singular terms

$$P_{gg}(z,k_\perp) = \bar{\alpha}_s\left[\left(\frac{(1-z)}{z} + \frac{z(1-z)}{2}\right)\Delta_{ns} + \left(\frac{z}{1-z} + \frac{z(1-z)}{2}\right)\right.$$

  - order in α_s : one or two loop ?

- kinematic constraints:

$$k_t'^{\,2} < z q_t^2$$
$$k_t'^{\,2} < \frac{z}{1-z} q_t^2$$
$$k_t'^{\,2} < z k_t^2$$

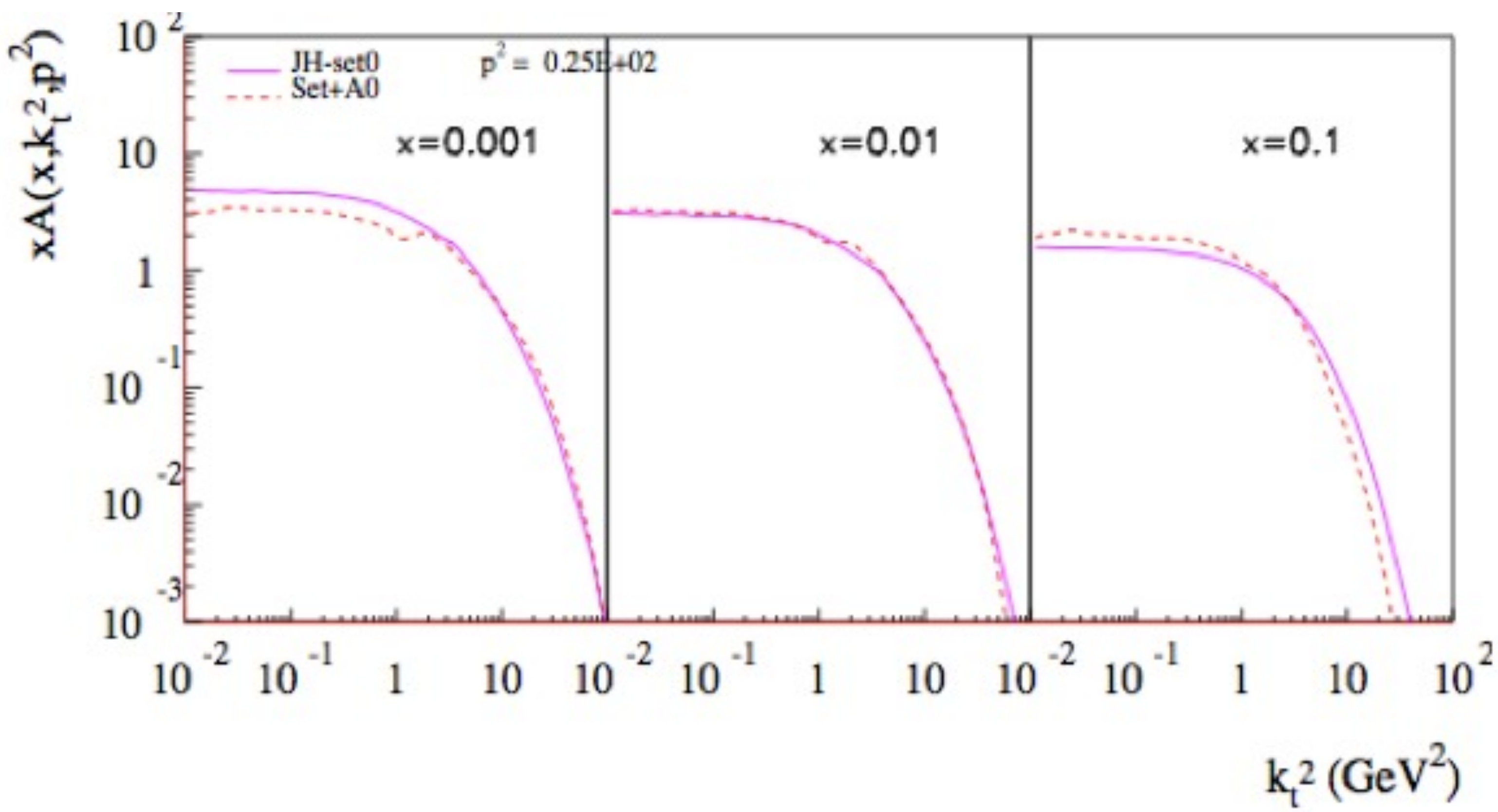


Results

- use H1/ZEUS combined NC data with: $Q^2 > 5\ \mathrm{GeV},\ x < 0.01$
- fitting 4 params (gluon only): $\mathcal{A}_0(x) = N_g x^{-B_g}(1-x)^{C_g}(1-D_g x)$
  - (consistency constraint)
    - no constraint (depending on splitting fct): $\chi^2/\mathrm{ndf} \sim 14\ldots28$
    - best results with: $k_t'^{\,2} < \frac{z}{1-z} q_t^2$ $\chi^2/\mathrm{ndf} \sim 1.7\ldots3.0$
  - Splitting function:
    - using singular terms (+kin constraint): $\chi^2/\mathrm{ndf} \sim 3$
    - non-singular terms (+kin constraint): $\chi^2/\mathrm{ndf} \sim 1.7$
- Final result (including valence quarks and treatment of correlated systematic uncertainties) for $Q^2 > 5\ \mathrm{GeV},\ x < 0.01$
  $\chi^2/\mathrm{ndf} \sim 1.6\ldots1.7$
  - depending on details of treatment of systematics:

Another question to investigate:

What effect would a saturation boundary in the evolution have on the fit?



Stop the evolution if k_t becomes smaller than Q_s (with possible damping)

$$Q_s^2 = \left(\frac{x_0}{x}\right)^{\lambda}$$

Early work by K. Kutak and H. Jung

study never completed, can be further investigated now.

Building uPDF's for ions

Define uPDF:

$$x\mathcal{G}(x,Q^2) \equiv \int^{Q^2} \mathrm{d}k^2 \frac{\mathcal{A}(x,k^2)}{k^2}$$

b-dependence!

Connection to dipole model:

$$\sigma_{q\bar{q}}(r,x) = \frac{8\pi^2}{N_C} \int \frac{\mathrm{d}k}{k^3}[1 - J_0(kr)]\alpha_{\mathrm{S}}\mathcal{A}(x,k^2) = 2\int \mathrm{d}^2\mathbf{b}\mathcal{N}(x,\mathbf{r},\mathbf{b})$$

uPDF from Scattering Amplitude:

$$\frac{\mathcal{A}(x,k^2)}{k^2} = \frac{C_F}{2\pi\alpha_{\mathrm{S}}(k)k^2} \int \mathrm{d}r\mathrm{d}bJ_0(rk)\left(\frac{1}{r}\frac{\partial\mathcal{N}(x,r,b)}{\partial r} + \frac{\partial^2\mathcal{N}(x,r,b)}{\partial r^2}\right)$$

Building uPDF's for ions

Building ion scattering amplitude:

$$1 - \mathcal{N}^{(A)} = \prod_{i=1}^{A} \left(1 - \mathcal{N}^{(p)}(x, r, |\mathbf{b} - \mathbf{b}_i|)\right)$$

Need:

A b-dependent proton scattering amplitude

Which:

Can be combined with the CCFM evolution in CASCADE (simple enough for the integrals to be analytically solved)

And:

Describes the HERA F_2 data

Building uPDF's for ions

Only option: GBW

$$\mathcal{N}(x,r) = \sigma_0 \left(1 - e^{-Q_s^2(x) r^2}\right)$$

with Θ function b-dependence (cylindrical proton):

$$T_p(b) = \Theta(R_p - b) \Rightarrow \sigma_0 = \pi R_p^2$$

Starting distribution in CASCADE:

$$x\mathcal{A}_0(x, k_t, q_0) = \frac{3\sigma_0}{4\pi^2 Q_s^2(x) \alpha_S(k_t)} k_t^2 e^{-\frac{k_t^2}{Q_s^2(x)}}$$

Parameters: x_0, λ, R_p

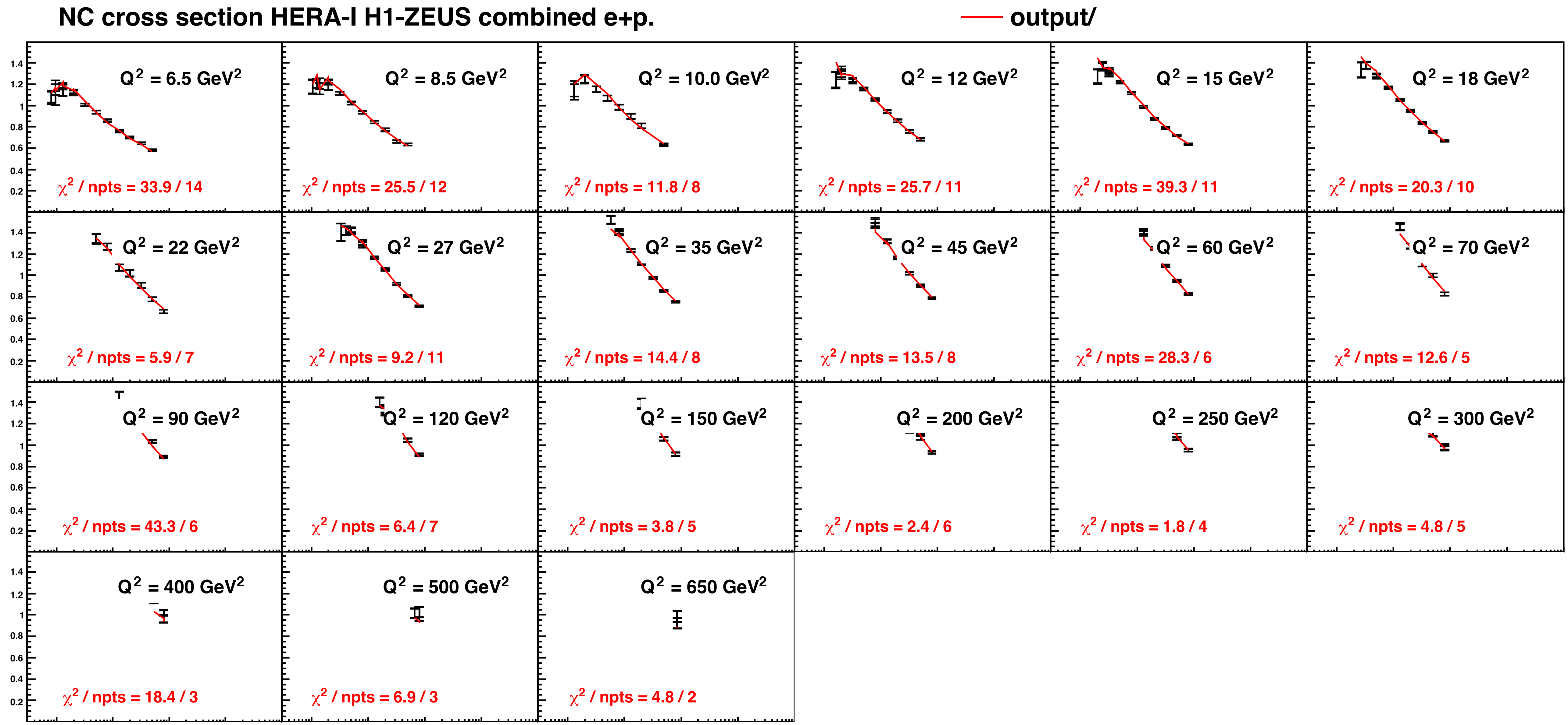
$$Q_s^2 = \left(\frac{x_0}{x}\right)^\lambda$$

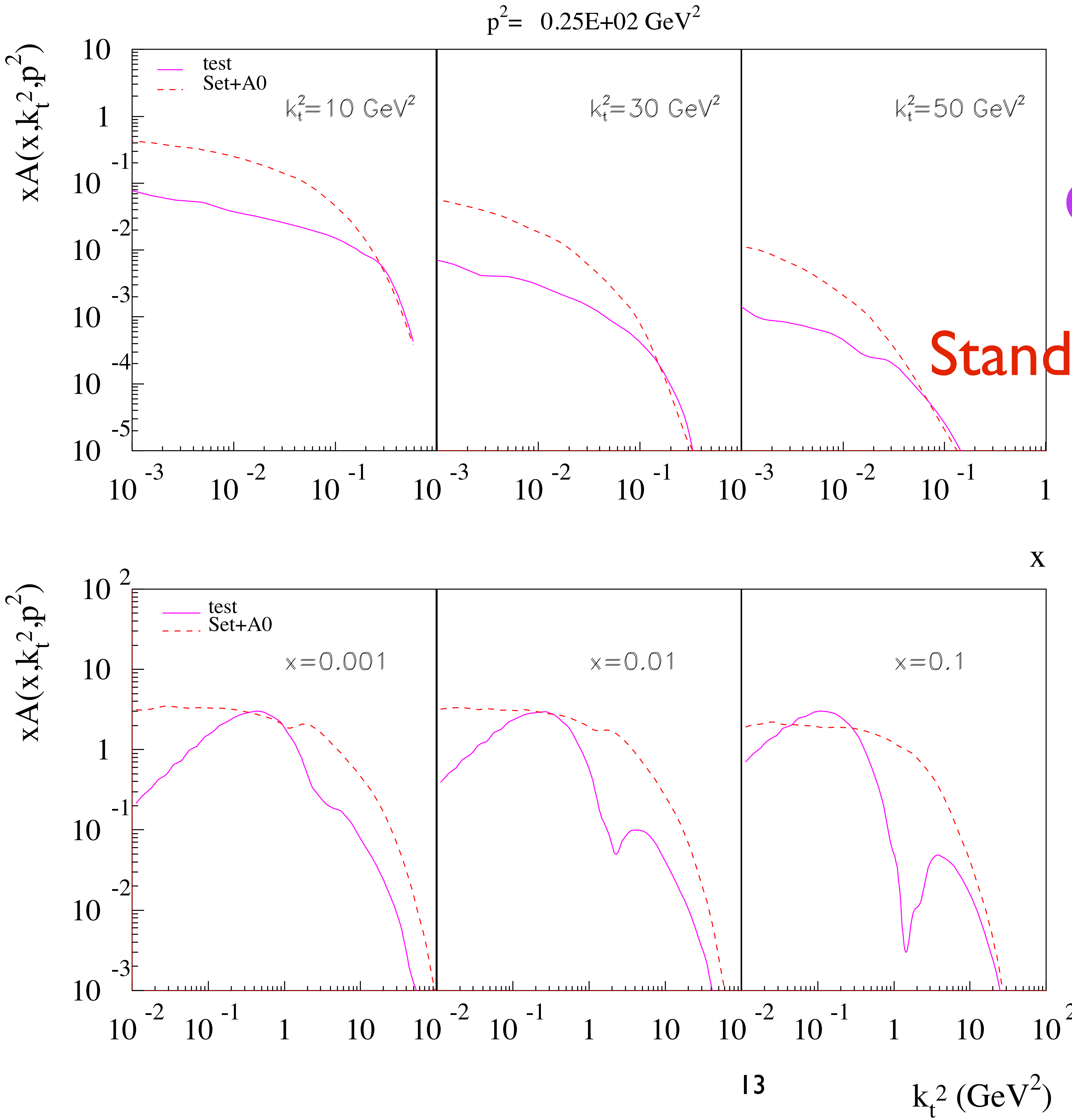
normalisation

Ongoing: Making fit with herafitter

Status:

	chi2/ndf	0.25	0.277	0.3
	4.10E-06	2.313	2.148	16.063
x0	4.10E-05	2.939	1.772	2.198
	4.10E-04	4.895	2.513	1.748





GBW uPDF

vs.

Standard CCFM uPDF

Next steps:

Use final fit to build ion uPDF's:

$$1-\mathcal{N}^{(A)}=\prod_{i=1}^{A}\left(1-\mathcal{N}^{(p)}(x,r,|\mathbf{b}-\mathbf{b}_i|)\right)$$

$$\frac{\mathcal{A}(x,k^2)}{k^2}=\frac{C_F}{2\pi\alpha_S(k)k^2}\int \mathrm{d}r\mathrm{d}b J_0(rk)\left(\frac{1}{r}\frac{\partial\mathcal{N}(x,r,b)}{\partial r}+\frac{\partial^2\mathcal{N}(x,r,b)}{\partial r^2}\right)$$

Investigate saturation effects in the evolution

Also: CASCADE uses JetSet for fragmentation

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we can use the same after-burner for nuclear effects as we want to use for PYTHIA